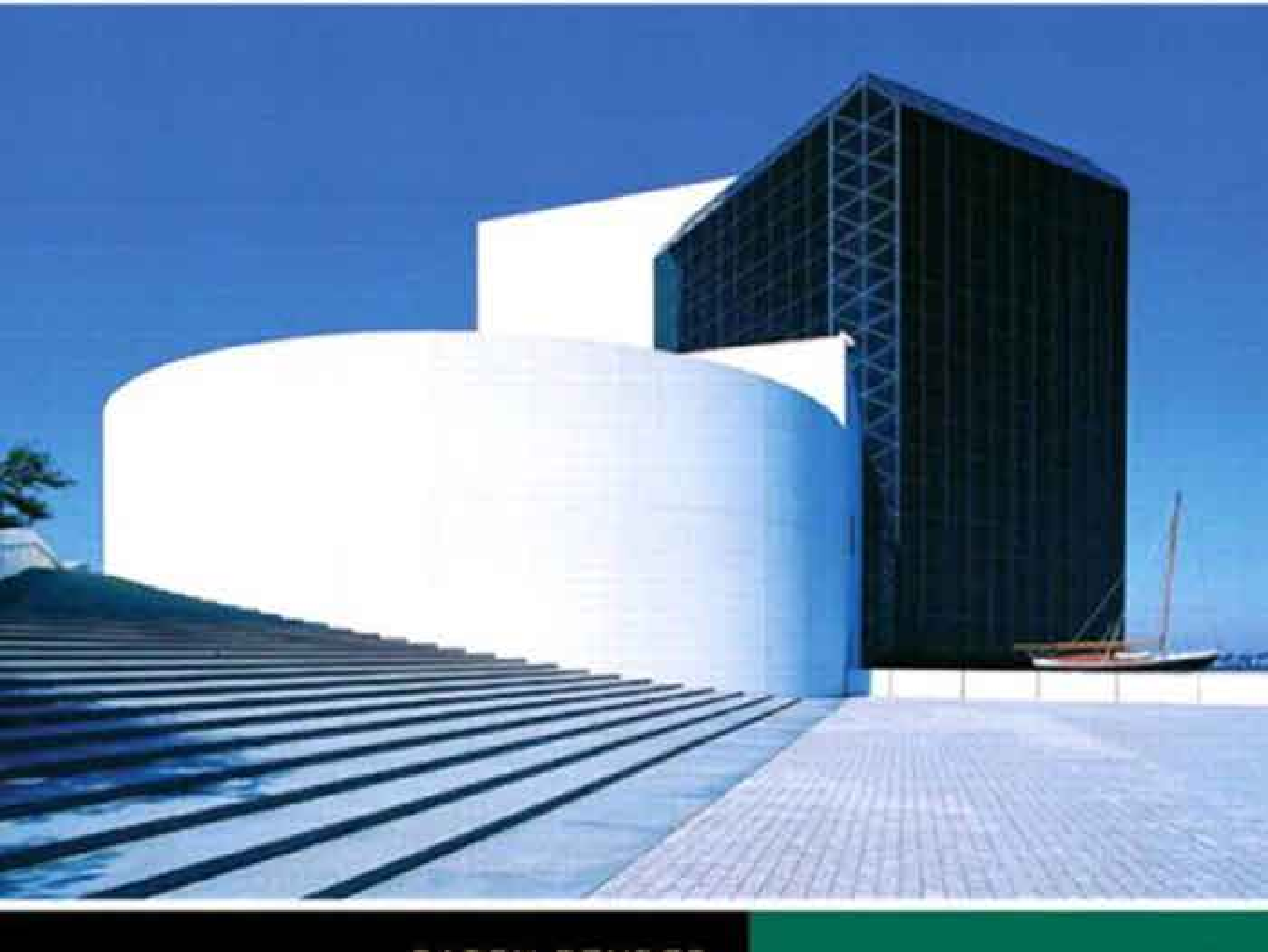
Quantitative Analysis for Management

Twelfth Edition



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TWELFTH EDITION

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Dr. Render has coauthored 10 textbooks published by Pearson, including *Managerial Decision Modeling with Spreadsheets*, *Operations Management*, *Principles of Operations Management*, *Service Management*, *Introduction to Management Science*, and *Cases and Readings in Management Science*. More than 100 articles of Dr. Render on a variety of management topics have appeared in *Decision Sciences*, *Production and Operations Management*, *Interfaces*, *Information and Management*, *Journal of Management Information Systems*, *Socio-Economic Planning Sciences*, *IIE Solutions,* and *Operations Management Review*, among others.

Dr. Render has been honored as an AACSB Fellow and was named twice as a Senior Fulbright Scholar. He was Vice President of the Decision Science Institute Southeast Region and served as software review editor for *Decision Line* for six years and as Editor of the *New York Times* Operations Management special issues for five years. From 1984 to 1993, Dr. Render was President of Management Service Associates of Virginia, Inc., whose technology clients included the FBI, the U.S. Navy, Fairfax County, Virginia, and C&P Telephone. He is currently Consulting Editor to *Financial Times Press*.

Dr. Render has taught operations management courses at Rollins College for MBA and Executive MBA programs. He has received that school's Welsh Award as leading professor and was selected by Roosevelt University as the 1996 recipient of the St. Claire Drake Award for Outstanding Scholarship. In 2005, Dr. Render received the Rollins College MBA Student Award for Best Overall Course, and in 2009 was named Professor of the Year by full-time MBA students.

Ralph Stair is Professor Emeritus at Florida State University. He earned a B.S. in chemical engineering from Purdue University and an M.B.A. from Tulane University. Under the guidance of Ken Ramsing and Alan Eliason, he received a Ph.D. in operations management from the University of Oregon. He has taught at the University of Oregon, the University of Washington, the University of New Orleans, and Florida State University.

He has taught twice in Florida State University's Study Abroad Program in London. Over the years, his teaching has been concentrated in the areas of information systems, operations research, and operations management.

Dr. Stair is a member of several academic organizations, including the Decision Sciences Institute and INFORMS, and he regularly participates in national meetings. He has published numerous articles and books, including *Managerial Decision Modeling with Spreadsheets, Introduction to Management Science, Cases and Readings in Management Science, Production and Operations Management: A Self-Correction Approach, Fundamentals of Information Systems, Principles of Information Systems, Introduction to Information Systems, Computers in Today's World, Principles* *of Data Processing, Learning to Live with Computers, Programming in BASIC, Essentials of BASIC Programming, Essentials of FORTRAN Programming,* and *Essentials of COBOL Programming.* Dr. Stair divides his time between Florida and Colorado. He enjoys skiing, biking, kayaking, and other outdoor activities.

Michael E. Hanna is Professor of Decision Sciences at the University of Houston–Clear Lake (UHCL). He holds a B.A. in Economics, an M.S. in Mathematics, and a Ph.D. in Operations Research from Texas Tech University. For more than 25 years, he has been teaching courses in statistics, management science, forecasting, and other quantitative methods. His dedication to teaching has been recognized with the Beta Alpha Psi teaching award in 1995 and the Outstanding Educator Award in 2006 from the Southwest Decision Sciences Institute (SWDSI).

Dr. Hanna has authored textbooks in management science and quantitative methods, has published numerous articles and professional papers, and has served on the Editorial Advisory Board of *Computers and Operations Research*. In 1996, the UHCL Chapter of Beta Gamma Sigma presented him with the Outstanding Scholar Award.

Dr. Hanna is very active in the Decision Sciences Institute, having served on the Innovative Education Committee, the Regional Advisory Committee, and the Nominating Committee. He has served on the board of directors of the Decision Sciences Institute (DSI) for two terms and also as regionally elected vice president of DSI. For SWDSI, he has held several positions, including president, and he received the SWDSI Distinguished Service Award in 1997. For overall service to the profession and to the university, he received the UHCL President's Distinguished Service Award in 2001.

Trevor S. Hale is Associate Professor of Management Science at the University of Houston–Downtown (UHD). He received a B.S. in Industrial Engineering from Penn State University, an M.S. in Engineering Management from Northeastern University, and a Ph.D. in Operations Research from Texas A&M University. He was previously on the faculty of both Ohio University–Athens, and Colorado State University–Pueblo.

Dr. Hale was honored three times as an Office of Naval Research Senior Faculty Fellow. He spent the summers of 2009, 2011, and 2013 performing energy security/cyber security research for the U.S. Navy at Naval Base Ventura County in Port Hueneme, California.

Dr. Hale has published dozens of articles in the areas of operations research and quantitative analysis in journals such as the *International Journal of Production Research*, the *European Journal of Operational Research*, *Annals of Operations Research*, the *Journal of the Operational Research Society,* and the *International Journal of Physical Distribution and Logistics Management* among several others. He teaches quantitative analysis courses in the University of Houston–Downtown MBA program and Masters of Security Management for Executives program. He is a senior member of both the Decision Sciences Institute and INFORMS.

BRIEF CONTENTS

CONTENTS

PREFACE

OVERVIEW

Welcome to the twelfth edition of *Quantitative Analysis for Management*. Our goal is to provide undergraduate and graduate students with a genuine foundation in business analytics, quantitative methods, and management science. In doing so, we owe thanks to the hundreds of users and scores of reviewers who have provided invaluable counsel and pedagogical insight for more than 30 years.

To help students connect how the techniques presented in this book apply in the real world, computer-based applications and examples are a major focus of this edition. Mathematical models, with all the necessary assumptions, are presented in a clear and "plain-English" manner. The ensuing solution procedures are then applied to example problems alongside step-by-step "how-to" instructions. We have found this method of presentation to be very effective and students are very appreciative of this approach. In places where the mathematical computations are intricate, the details are presented in such a manner that the instructor can omit these sections without interrupting the flow of material. The use of computer software enables the instructor to focus on the managerial problem and spend less time on the details of the algorithms. Computer output is provided for many examples throughout the book.

The only mathematical prerequisite for this textbook is algebra. One chapter on probability and another on regression analysis provide introductory coverage on these topics. We employ standard notation, terminology, and equations throughout the book. Careful explanation is provided for the mathematical notation and equations that are used.

NEW TO THIS EDITION

- An introduction to business analytics is provided.
- Excel 2013 is incorporated throughout the chapters.
- The transportation, assignment, and network models have been combined into one chapter focused on modeling with linear programming.
- Specialized algorithms for the transportation, assignment, and network methods have been combined into Online Module 8.
- New examples, over 25 problems, 8 QA in Action applications, 4 Modeling in the Real World features, and 3 new Case Studies have been added throughout the textbook. Other problems and Case Studies have been updated.

SPECIAL FEATURES

Many features have been popular in previous editions of this textbook, and they have been updated and expanded in this edition. They include the following:

- *Modeling in the Real World* boxes demonstrate the application of the quantitative analysis approach to every technique discussed in the book. Four new ones have been added.
- *Procedure* boxes summarize the more complex quantitative techniques, presenting them as a series of easily understandable steps.
- *Margin notes* highlight the important topics in the text.
- *History* boxes provide interesting asides related to the development of techniques and the people who originated them.
- *QA in Action* boxes illustrate how real organizations have used quantitative analysis to solve problems. Several new QA in Action boxes have been added.
- *Solved Problems*, included at the end of each chapter, serve as models for students in solving their own homework problems.
- *Discussion Questions* are presented at the end of each chapter to test the student's understanding of the concepts covered and definitions provided in the chapter.
- *Problems* included in every chapter are applications oriented and test the student's ability to solve exam-type problems. They are graded by level of difficulty: introductory (one bullet), moderate (two bullets), and challenging (three bullets). More than 40 new problems have been added.
- *Internet Homework Problems* provide additional problems for students to work. They are available on the Companion Website.
- *Self-Tests* allow students to test their knowledge of important terms and concepts in preparation for quizzes and examinations.
- *Case Studies*, at the end of each chapter, provide additional challenging managerial applications.
- *Glossaries*, at the end of each chapter, define important terms.
- *Key Equations*, provided at the end of each chapter, list the equations presented in that chapter.
- *End-of-chapter bibliographies* provide a current selection of more advanced books and articles.
- *The software POM-QM for Windows* uses the full capabilities of Windows to solve quantitative analysis problems.
- *Excel QM* and *Excel 2013* are used to solve problems throughout the book.
- Data files with Excel spreadsheets and POM-QM for Windows files containing all the examples in the textbook are available for students to download from the Companion Website. Instructors can download these plus additional files containing computer solutions to the relevant end-of-chapter problems from the Instructor Resource Center Web site.
- *Online modules* provide additional coverage of topics in quantitative analysis.
- The Companion Website, at www.pearsonhighered.com/render, provides the online modules, additional problems, cases, and other material for almost every chapter.

SIGNIFICANT CHANGES TO THE TWELFTH EDITION

In the twelfth edition, we have introduced Excel 2013 in all of the chapters. Screenshots are integrated in the appropriate sections so that students can easily learn how to use Excel for the calculations. The Excel QM add-in is used with Excel 2013 allowing students with limited Excel experience to easily perform the necessary calculations. This also allows students to improve their Excel skills as they see the formulas automatically written in Excel QM.

From the Companion Website, students can access files for all of the examples used in the textbook in Excel 2013, QM for Windows, and Excel QM. Other files with all of the end-of-chapter problems involving these software tools are available to the instructors.

Business analytics, one of the hottest topics in the business world, makes extensive use of the models in this book. A discussion of the business analytics categories is provided, and the relevant management science techniques are placed into the appropriate category.

The transportation, transshipment, assignment, and network models have been combined into one chapter focused on modeling with linear programming. The specialized algorithms for these models have been combined into a new online module.

Examples and problems have been updated, and many new ones have been added. New screenshots are provided for almost all of the examples in the book. A brief summary of the other changes in each chapter are presented here.

Chapter 1 *Introduction to Quantitative Analysis*. A section on business analytics has been added, the self-test has been modified, and two new problems were added.

Chapter 2 *Probability Concepts and Applications.* The presentation of the fundamental concepts of probability has been significantly modified and reorganized. Two new problems have been added.

Chapter 3 *Decision Analysis.* A more thorough discussion of minimization problems with payoff tables has been provided in a new section. The presentation of software usage with payoff tables was expanded. Two new problems were added.

Chapter 4 *Regression Models.* The use of different software packages for regression analysis has been moved to the body of the textbook instead of the appendix. Five new problems and one new QA in Action item have been added.

Chapter 5 *Forecasting*. The presentation of time-series forecasting models was significantly revised to bring the focus on identifying the appropriate technique to use based on which time-series components are present in the data. Five new problems were added, and the cases have been updated.

Chapter 6 *Inventory Control Models*. The four steps of the Kanban production process have been updated and clarified. Two new QA in Action boxes, four new problems, and one new Modeling in the Real World have been added.

Chapter 7 *Linear Programming Models: Graphical and Computer Methods.* More discussion of Solver is presented. A new Modeling in the Real World item was added, and the solved problems have been revised.

Chapter 8 *Linear Programming Applications.* The transportation model was moved to Chapter 9, and a new section describing other models has been added. The self-test questions were modified; one new problem, one new QA in Action summary, and a new case study have been added.

Chapter 9 *Transportation, Assignment, and Network Models*. This new chapter presents all of the distribution, assignment, and network models that were previously in two separate chapters. The modeling approach is emphasized, while the special-purpose algorithms were moved to a new online module. A new case study, Northeastern Airlines, has also been added.

Chapter 10 *Integer Programming, Goal Programming, and Nonlinear Programming.* The use of Excel 2013 and the new screen shots were the only changes to this chapter.

Chapter 11 *Project Management*. Two new end-of-chapter problems and three new QA in Action boxes have been added.

Chapter 12 *Waiting Lines and Queuing Theory Models*. Two new end-of-chapter problems were added.

Chapter 13 *Simulation Modeling*. One new Modeling in the Real World vignette, one new QA in Action box, and a new case study have been added.

Chapter 14 *Markov Analysis*. One new QA in Action box and two new end-of-chapter problems have been added.

Chapter 15 *Statistical Quality Control*. One new Modeling in the Real World vignette, one new QA in Action box, and two new end-of-chapter problems have been added.

Modules 1–8 The only significant change to the modules is the addition of Module 8: *Transportation, Assignment, and Network Algorithms*. This includes the special-purpose algorithms for the transportation, assignment, and network models.

ONLINE MODULES

To streamline the book, eight topics are contained in modules available on the Companion Website for the book.

1. Analytic Hierarchy Process
2. Dynamic Programming
3. Decision Theory and the Normal Distribution
4. Game Theory
5. Mathematical Tools: Determinants and Matrices
6. Calculus-Based Optimization
7. Linear Programming: The Simplex Method
8. Transportation, Assignment, and Network Algorithms

SOFTWARE

Excel 2013 Instructions and screen captures are provided for, using Excel 2013, throughout the book. Instructions for activating the Solver and Analysis ToolPak add-ins in Excel 2013 are provided in an appendix. The use of Excel is more prevalent in this edition of the book than in previous editions.

Excel QM Using the Excel QM add-in that is available on the Companion Website makes the use of Excel even easier. Students with limited Excel experience can use this and learn from the formulas that are automatically provided by Excel QM. This is used in many of the chapters.

POM-QM for Windows This software, developed by Professor Howard Weiss, is available to students at the Companion Website. This is very user-friendly and has proven to be a very popular software tool for users of this textbook. Modules are available for every major problem type presented in the textbook.

COMPANION WEBSITE

The Companion Website, located at www.pearsonhighered.com/render, contains a variety of materials to help students master the material in this course. These include the following:

Modules There are eight modules containing additional material that the instructor may choose to include in the course. Students can download these from the Companion Website.

Files for Examples in Excel, Excel QM, and POM-QM for Windows Students can download the files that were used for examples throughout the book. This helps them become familiar with the software, and it helps them understand the input and formulas necessary for working the examples.

Internet Homework Problems In addition to the end-of-chapter problems in the textbook, there are additional problems that instructors may assign. These are available for download at the Companion Website.

Internet Case Studies Additional case studies are available for most chapters.

POM-QM for Windows Developed by Howard Weiss, this very user-friendly software can be used to solve most of the homework problems in the text.

Excel QM This Excel add-in will automatically create worksheets for solving problems. This is very helpful for instructors who choose to use Excel in their classes but who may have students with limited Excel experience. Students can learn by examining the formulas that have been created, and by seeing the inputs that are automatically generated for using the Solver add-in for linear programming.

INSTRUCTOR RESOURCES

- ***Instructor Resource Center:*** The Instructor Resource Center contains the electronic files for the test bank, PowerPoint slides, the Solutions Manual, and data files for both Excel and POM-QM for Windows for all relevant examples and end-of-chapter problems. (www.pearsonhighered.com/render).
- ***Register, Redeem, Login:*** At www.pearsonhighered.com/irc, instructors can access a variety of print, media, and presentation resources that are available with this text in downloadable, digital format. For most texts, resources are also available for course management platforms such as Blackboard, WebCT, and Course Compass.
- ***Need help?*** Our dedicated technical support team is ready to assist instructors with questions about the media supplements that accompany this text. Visit http://247pearsoned.custhelp.com/ for answers to frequently asked questions and toll-free user support phone numbers. The supplements are available to adopting instructors. Detailed descriptions are provided on the Instructor Resource Center.

Instructor's Solutions Manual The Instructor's Solutions Manual, updated by the authors, is available for download from the Instructor Resource Center. Solutions to all Internet Homework Problems and Internet Case Studies are also included in the manual.

PowerPoint Presentation An extensive set of PowerPoint slides is available for download from the Instructor Resource Center.

Test Bank The updated test bank is available for download from the Instructor Resource Center.

TestGen The computerized TestGen package allows instructors to customize, save, and generate classroom tests. The test program permits instructors to edit, add, or delete questions from the test bank; edit existing graphics and create new graphics; analyze test results; and organize a database of test and student results. This software allows the instructors to benefit from the extensive flexibility and ease of use. It provides many options for organizing and displaying tests, along with search and sort features. The software and the test banks can be downloaded at www.pearsonhighered.com/render.

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We thank Howard Weiss for providing Excel QM and POM-QM for Windows, two of the most outstanding packages in the field of quantitative methods. We would also like to thank the reviewers who have helped to make this textbook the most widely used one in the field of quantitative analysis:

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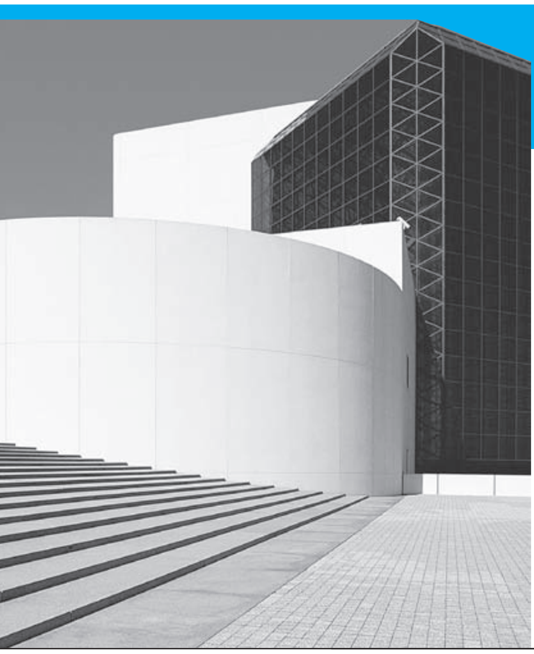


CHAPTER 1

Introduction to Quantitative Analysis

LEARNING OBJECTIVES

After completing this chapter, students will be able to:

1. Describe the quantitative analysis approach.
2. Understand the application of quantitative analysis in a real situation.
3. Describe the three categories of business analytics.
4. Describe the use of modeling in quantitative analysis.
5. Use computers and spreadsheet models to perform quantitative analysis.
6. Discuss possible problems in using quantitative analysis.
7. Perform a break-even analysis.

CHAPTER OUTLINE

1.1 Introduction

People have been using mathematical tools to help solve problems for thousands of years; however, the formal study and application of quantitative techniques to practical decision making is largely a product of the twentieth century. The techniques we study in this book have been applied successfully to an increasingly wide variety of complex problems in business, government, health care, education, and many other areas. Many such successful uses are discussed throughout this book.

It isn't enough, though, just to know the mathematics of how a particular quantitative technique works; you must also be familiar with the limitations, assumptions, and specific applicability of the technique. The successful use of quantitative techniques usually results in a solution that is timely, accurate, flexible, economical, reliable, and easy to understand and use.

In this and other chapters, there are *QA (Quantitative Analysis) in Action* boxes that provide success stories on the applications of management science. They show how organizations have used quantitative techniques to make better decisions, operate more efficiently, and generate more profits. Taco Bell has reported saving over $150 million with better forecasting of demand and better scheduling of employees. NBC television increased advertising revenue by over $200 million between 1996 and 2000 by using a model to help develop sales plans for advertisers. Continental Airlines saves over $40 million per year by using mathematical models to quickly recover from disruptions caused by weather delays and other factors. These are but a few of the many companies discussed in *QA in Action* boxes throughout this book.

To see other examples of how companies use quantitative analysis or operations research methods to operate better and more efficiently, go to the website www.scienceofbetter.org. The success stories presented there are categorized by industry, functional area, and benefit. These success stories illustrate how operations research is truly the "science of better."

1.2 What Is Quantitative Analysis?

Quantitative analysis uses a scientific approach to decision making.

Quantitative analysis is the scientific approach to managerial decision making. This field of study has several different names including quantitative analysis, **management science**, and operations research. These terms are used interchangeably in this book. Also, many of the quantitative analysis methods presented in this book are used extensively in business analytics.

Whim, emotions, and guesswork are not part of the quantitative analysis approach. The approach starts with data. Like raw material for a factory, these data are manipulated or processed into information that is valuable to people making decisions. This processing and manipulating of raw data into meaningful information is the heart of quantitative analysis. Computers have been instrumental in the increasing use of quantitative analysis.

In solving a problem, managers must consider both qualitative and quantitative factors. For example, we might consider several different investment alternatives, including certificates of deposit at a bank, investments in the stock market, and an investment in real estate. We can use quantitative analysis to determine how much our investment will be worth in the future when deposited at a bank at a given interest rate for a certain number of years. Quantitative analysis can also be used in computing financial ratios from the balance sheets for several companies whose stock we are considering. Some real estate companies have developed computer programs that use quantitative analysis to analyze cash flows and rates of return for investment property.

Both qualitative and quantitative factors must be considered.

In addition to quantitative analysis, *qualitative* factors should also be considered. The weather, state and federal legislation, new technological breakthroughs, the outcome of an election, and so on may all be factors that are difficult to quantify.

Because of the importance of qualitative factors, the role of quantitative analysis in the decision-making process can vary. When there is a lack of qualitative factors and when the problem, model, and input data remain the same, the results of quantitative analysis can *automate* the decision-making process. For example, some companies use quantitative inventory models to determine automatically *when* to order additional new materials. In most cases, however, quantitative analysis will be an *aid* to the decision-making process. The results of quantitative analysis will be combined with other (qualitative) information in making decisions.

Quantitative analysis has been particularly important in many areas of management. The field of production management, which evolved into production/operations management (POM) as society became more service oriented, uses quantitative analysis extensively. While POM focuses on internal operations of a company, the field of supply chain management takes a more complete view of the business and considers the entire process of obtaining materials from suppliers, using the materials to develop products, and distributing these products to the final consumers. Supply chain management makes extensive use of many management science models. Another area of management that could not exist without the quantitative analysis methods presented in this book, and perhaps the hottest discipline in business today, is business analytics.

1.3 Business Analytics

Business analytics is a data-driven approach to decision making that allows companies to make better decisions. The study of business analytics involves the use of large amounts of data, which means that information technology related to the management of the data is very important. Statistical and quantitative analysis are used to analyze the data and provide useful information to the decision maker.

Business analytics is often broken into three categories: descriptive, predictive, and prescriptive. **Descriptive analytics** involves the study and consolidation of historical data for a business and an industry. It helps a company measure how it has performed in the past and how it is performing now. **Predictive analytics** is aimed at forecasting future outcomes based on patterns in the past data. Statistical and mathematical models are used extensively for this purpose. **Prescriptive analytics** involves the use of optimization methods to provide new and better ways to operate based on specific business objectives. The optimization models presented in this book are very important to prescriptive analytics. While there are only three business analytics categories, many business decisions are made based on information obtained from two or three of these categories.

Many of the quantitative analysis techniques presented in the chapters of this book are used extensively in business analytics. Table 1.1 highlights the three categories of business analytics, and it places many of the topics and chapters in this book in the most relevant category. Keep in mind that some topics (and certainly some chapters with multiple concepts and models) could possibly be placed in a different category. Some of the material in this book could overlap two or even three of these categories. Nevertheless, all of these quantitative analysis techniques are very important tools in business analytics.

The three categories of business analytics are descriptive, predictive, and prescriptive.

TABLE 1.1
Business Analytics and Quantitative Analysis Models

BUSINESS ANALYTICS CATEGORY	QUANTITATIVE ANALYSIS TECHNIQUE (CHAPTER)
Descriptive analytics	• Statistical measures such as means and standard deviations (Chapter 2) • Statistical quality control (Chapter 15)
Predictive analytics	• Decision analysis and decision trees (Chapter 3) • Regression models (Chapter 4) • Forecasting (Chapter 5) • Project scheduling (Chapter 11) • Waiting line models (Chapter 12) • Simulation (Chapter 13) • Markov analysis (Chapter 14)
Prescriptive analytics	• Inventory models such as the economic order quantity (Chapter 6) • Linear programming (Chapters 7, 8) • Transportation and assignment models (Chapter 9) • Integer programming, goal programming, and nonlinear programming (Chapter 10) • Network models (Chapter 9)

HISTORY The Origin of Quantitative Analysis

Quantitative analysis has been in existence since the beginning of recorded history, but it was Frederick W. Taylor who in the early 1900s pioneered the principles of the scientific approach to management. During World War II, many new scientific and quantitative techniques were developed to assist the military. These new developments were so successful that after World War II many companies started using similar techniques in managerial decision making and planning. Today, many organizations employ a staff of operations research or management science personnel or consultants to apply the principles of scientific management to problems and opportunities.

The origin of many of the techniques discussed in this book can be traced to individuals and organizations that have applied the principles of scientific management first developed by Taylor; they are discussed in *History* boxes scattered throughout the book.

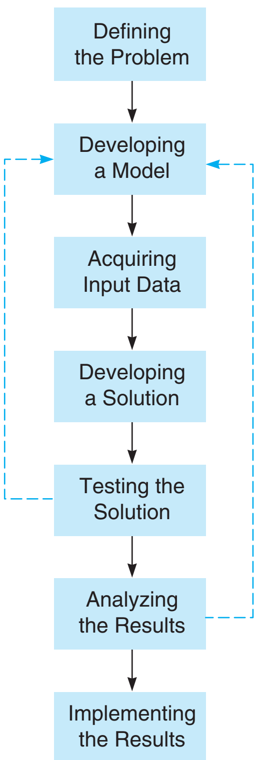
1.4 The Quantitative Analysis Approach

Defining the problem can be the most important step.

Concentrate on only a few problems.

The quantitative analysis approach consists of defining a problem, developing a model, acquiring input data, developing a solution, testing the solution, analyzing the results, and implementing the results (see Figure 1.1). One step does not have to be finished completely before the next is started; in most cases, one or more of these steps will be modified to some extent before the final results are implemented. This would cause all of the subsequent steps to be changed. In some cases, testing the solution might reveal that the model or the input data are not correct. This would mean that all steps that follow defining the problem would need to be modified.

FIGURE 1.1
The Quantitative Analysis Approach



Defining the Problem

The first step in the quantitative approach is to develop a clear, concise statement of the **problem**. This statement will give direction and meaning to the following steps.

In many cases, defining the problem is the most important and the most difficult step. It is essential to go beyond the symptoms of the problem and identify the true causes. One problem may be related to other problems; solving one problem without regard to other related problems can make the entire situation worse. Thus, it is important to analyze how the solution to one problem affects other problems or the situation in general.

It is likely that an organization will have several problems. However, a quantitative analysis group usually cannot deal with all of an organization's problems at one time. Thus, it is usually necessary to concentrate on only a few problems. For most companies, this means selecting those problems whose solutions will result in the greatest increase in profits or reduction in costs to the company. The importance of selecting the right problems to solve cannot be overemphasized. Experience has shown that bad problem definition is a major reason for failure of management science or operations research groups to serve their organizations well.

When the problem is difficult to quantify, it may be necessary to develop *specific*, *measurable* objectives. A problem might be inadequate health care delivery in a hospital. The objectives might be to increase the number of beds, reduce the average number of days a patient spends in the hospital, increase the physician-to-patient ratio, and so on. When objectives are used, however, the real problem should be kept in mind. It is important to avoid obtaining specific and measurable objectives that may not solve the real problem.

Developing a Model

Once we select the problem to be analyzed, the next step is to develop a **model**. Simply stated, a model is a representation (usually mathematical) of a situation.

Even though you might not have been aware of it, you have been using models most of your life. You may have developed models about people's behavior. Your model might be that friendship is based on reciprocity, an exchange of favors. If you need a favor such as a small loan, your model would suggest that you ask a good friend.

The types of models include physical, scale, schematic, and mathematical models.

Of course, there are many other types of models. Architects sometimes make a *physical model* of a building that they will construct. Engineers develop *scale models* of chemical plants, called pilot plants. A *schematic model* is a picture, drawing, or chart of reality. Automobiles, lawn mowers, gears, fans, typewriters, and numerous other devices have schematic models (drawings and

IN ACTION Operations Research and Oil Spills

Operations researchers and decision scientists have been investigating oil spill response and alleviation strategies since long before the BP oil spill disaster of 2010 in the Gulf of Mexico. A four-phase classification system has emerged for disaster response research: mitigation, preparedness, response, and recovery. *Mitigation* means reducing the probability that a disaster will occur and implementing robust, forward-thinking strategies to reduce the effects of a disaster that does occur. *Preparedness* is any and all organization efforts that happen *a priori* to a disaster. *Response* is the location, allocation, and overall coordination of resources and procedures during the disaster that are aimed at preserving life and property. *Recovery* is the set of actions taken to minimize the long-term impacts of a particular disaster after the immediate situation has stabilized.

Many quantitative tools have helped in areas of risk analysis, insurance, logistical preparation and supply management, evacuation planning, and development of communication systems. Recent research has shown that while many strides and discoveries have been made, much research is still needed. Certainly each of the four disaster response areas could benefit from additional research, but recovery seems to be of particular concern and perhaps the most promising for future research.

Source: Based on N. Altay and W. Green. "OR/MS Research in Disaster Operations Management," *European Journal of Operational Research* 175, 1 (2006): 475–493.

pictures) that reveal how these devices work. What sets quantitative analysis apart from other techniques is that the models that are used are mathematical. A **mathematical model** is a set of mathematical relationships. In most cases, these relationships are expressed in equations and inequalities, as they are in a spreadsheet model that computes sums, averages, or standard deviations.

Although there is considerable flexibility in the development of models, most of the models presented in this book contain one or more variables and parameters. A **variable**, as the name implies, is a measurable quantity that may vary or is subject to change. Variables can be *controllable* or *uncontrollable*. A controllable variable is also called a *decision variable*. An example would be how many inventory items to order. A **parameter** is a measurable quantity that is inherent in the problem. The cost of placing an order for more inventory items is an example of a parameter. In most cases, variables are unknown quantities, while parameters are known quantities. All models should be developed carefully. They should be solvable, realistic, and easy to understand and modify, and the required **input data** should be obtainable. The model developer has to be careful to include the appropriate amount of detail to be solvable yet realistic.

Acquiring Input Data

Garbage in, garbage out means that improper data will result in misleading results.

Once we have developed a model, we must obtain the data that are used in the model (*input data*). Obtaining accurate data for the model is essential; even if the model is a perfect representation of reality, improper data will result in misleading results. This situation is called *garbage in, garbage out*. For a larger problem, collecting accurate data can be one of the most difficult steps in performing quantitative analysis.

There are a number of sources that can be used in collecting data. In some cases, company reports and documents can be used to obtain the necessary data. Another source is interviews with employees or other persons related to the firm. These individuals can sometimes provide excellent information, and their experience and judgment can be invaluable. A production supervisor, for example, might be able to tell you with a great degree of accuracy the amount of time it takes to produce a particular product. Sampling and direct measurement provide other sources of data for the model. You may need to know how many pounds of raw material are used in producing a new photochemical product. This information can be obtained by going to the plant and actually measuring with scales the amount of raw material that is being used. In other cases, statistical sampling procedures can be used to obtain data.

Developing a Solution

Developing a solution involves manipulating the model to arrive at the best (optimal) solution to the problem. In some cases, this requires that an equation be solved for the best decision. In other cases, you can use a *trial and error* method, trying various approaches and picking the one that results in the best decision. For some problems, you may wish to try all possible values for the variables in the model to arrive at the best decision. This is called *complete enumeration*. This book also shows you how to solve very difficult and complex problems by repeating a few simple steps until you find the best solution. A series of steps or procedures that are repeated is called an **algorithm**, named after Algorismus, an Arabic mathematician of the ninth century.

The input data and model determine the accuracy of the solution.

The accuracy of a solution depends on the accuracy of the input data and the model. If the input data are accurate to only two significant digits, then the results can be accurate to only two significant digits. For example, the results of dividing 2.6 by 1.4 should be 1.9, not 1.857142857.

Testing the Solution

Testing the data and model is done before the results are analyzed.

Before a solution can be analyzed and implemented, it needs to be tested completely. Because the solution depends on the input data and the model, both require testing.

Testing the input data and the model includes determining the accuracy and completeness of the data used by the model. Inaccurate data will lead to an inaccurate solution. There are several ways to test input data. One method of testing the data is to collect additional data from a different source. If the original data were collected using interviews, perhaps some additional data can be collected by direct measurement or sampling. These additional data can then be compared with the original data, and statistical tests can be employed to determine whether there are differences between the original data and the additional data. If there are significant differences, more effort is required to obtain accurate input data. If the data are accurate but the results are inconsistent with the problem, the model may not be appropriate. The model can be checked to make sure that it is logical and represents the real situation.

Although most of the quantitative techniques discussed in this book have been computerized, you will probably be required to solve a number of problems by hand. To help detect both logical and computational mistakes, you should check the results to make sure that they are consistent with the structure of the problem. For example, (1.96)(301.7) is close to (2)(300), which is equal to 600. If your computations are significantly different from 600, you know you have made a mistake.

Analyzing the Results and Sensitivity Analysis

Analyzing the results starts with determining the implications of the solution. In most cases, a solution to a problem will result in some kind of action or change in the way an organization is operating. The implications of these actions or changes must be determined and analyzed before the results are implemented.

Sensitivity analysis determines how the solutions will change with a different model or input data.

Because a model is only an approximation of reality, the sensitivity of the solution to changes in the model and input data is a very important part of analyzing the results. This type of analysis is called **sensitivity analysis** or *postoptimality analysis*. It determines how much the solution will change if there were changes in the model or the input data. When the solution is sensitive to changes in the input data and the model specification, additional testing should be performed to make sure that the model and input data are accurate and valid. If the model or data are wrong, the solution could be wrong, resulting in financial losses or reduced profits.

The importance of sensitivity analysis cannot be overemphasized. Because input data may not always be accurate or model assumptions may not be completely appropriate, sensitivity analysis can become an important part of the quantitative analysis approach. Most of the chapters in the book cover the use of sensitivity analysis as part of the decision-making and problem-solving process.

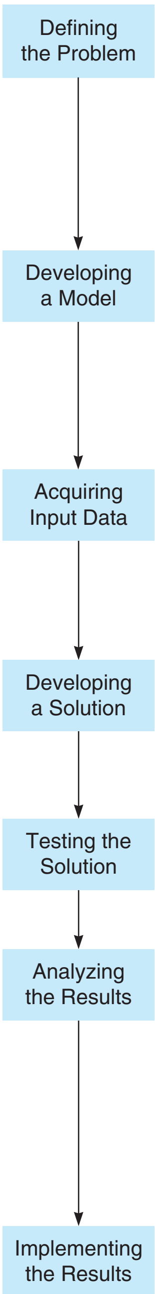
Implementing the Results

The final step is to *implement* the results. This is the process of incorporating the solution into the company. This can be much more difficult than you would imagine. Even if the solution is optimal and will result in millions of dollars in additional profits, if managers resist the new solution, all of the efforts of the analysis are of no value. Experience has shown that a large number of quantitative analysis teams have failed in their efforts because they have failed to implement a good, workable solution properly.

After the solution has been implemented, it should be closely monitored. Over time, there may be numerous changes that call for modifications of the original solution. A changing economy, fluctuating demand, and model enhancements requested by managers and decision makers are only a few examples of changes that might require the analysis to be modified.

MODELING IN THE REAL WORLD

Railroad Uses Optimization Models to Save Millions



Defining the Problem

CSX Transportation, Inc., has 35,000 employees and annual revenue of $11 billion. It provides rail freight services to 23 states east of the Mississippi River, as well as parts of Canada. CSX receives orders for rail delivery service and must send empty railcars to customer locations. Moving these empty railcars results in hundreds of thousands of empty-car miles every day. If allocations of railcars to customers is not done properly, problems arise from excess costs, wear and tear on the system, and congestion on the tracks and at rail yards.

Developing a Model

In order to provide a more efficient scheduling system, CSX spent 2 years and $5 million developing its Dynamic Car-Planning (DCP) system. This model will minimize costs, including car travel distance, car handling costs at the rail yards, car travel time, and costs for being early or late. It does this while at the same time filling all orders, making sure the right type of car is assigned to the job, and getting the car to the destination in the allowable time.

Acquiring Input Data

In developing the model, the company used historical data for testing. In running the model, the DCP uses three external sources to obtain information on the customer car orders, the available cars of the type needed, and the transit-time standards. In addition to these, two internal input sources provide information on customer priorities and preferences and on cost parameters.

Developing a Solution

This model takes about 1 minute to load but only 10 seconds to solve. Because supply and demand are constantly changing, the model is run about every 15 minutes. This allows final decisions to be delayed until absolutely necessary.

Testing the Solution

The model was validated and verified using existing data. The solutions found using the DCP were found to be very good compared to assignments made without DCP.

Analyzing the Results

Since the implementation of DCP in 1997, more than $51 million has been saved annually. Due to the improved efficiency, it is estimated that CSX avoided spending another $1.4 billion to purchase an additional 18,000 railcars that would have been needed without DCP. Other benefits include reduced congestion in the rail yards and reduced congestion on the tracks, which are major concerns. This greater efficiency means that more freight can ship by rail rather than by truck, resulting in significant public benefits. These benefits include reduced pollution and greenhouse gases, improved highway safety, and reduced road maintenance costs.

Implementing the Results

Both senior-level management who championed DCP and key car-distribution experts who supported the new approach were instrumental in gaining acceptance of the new system and overcoming problems during the implementation. The job description of the car distributors was changed from car allocators to cost technicians. They are responsible for seeing that accurate cost information is entered into DCP, and they also manage any exceptions that must be made. They were given extensive training on how DCP works so they could understand and better accept the new system. Due to the success of DCP, other railroads have implemented similar systems and achieved similar benefits. CSX continues to enhance DCP to make DCP even more customer friendly and to improve car-order forecasts.

Source: Based on M. F. Gorman, et al. "CSX Railway Uses OR to Cash in on Optimized Equipment Distribution," *Interfaces* 40, 1 (January–February 2010): 5–16.

The Quantitative Analysis Approach and Modeling in the Real World

The quantitative analysis approach is used extensively in the real world. These steps, first seen in Figure 1.1 and described in this section, are the building blocks of any successful use of quantitative analysis. As seen in our first *Modeling in the Real World* box, the steps of the quantitative analysis approach can be used to help a large company such as CSX plan for critical scheduling needs now and for decades into the future. Throughout this book, you will see how the steps of the quantitative analysis approach are used to help countries and companies of all sizes save millions of dollars, plan for the future, increase revenues, and provide higher-quality products and services. The *Modeling in the Real World* boxes in every chapter will demonstrate to you the power and importance of quantitative analysis in solving real problems for real organizations. Using the steps of quantitative analysis, however, does not guarantee success. These steps must be applied carefully.

1.5 How to Develop a Quantitative Analysis Model

Developing a model is an important part of the quantitative analysis approach. Let's see how we can use the following mathematical model, which represents profit:

$$\text{Profit} = \text{Revenue} - \text{Expenses}$$

Expenses include fixed and variable costs.

In many cases, we can express revenues as price per unit multiplied times the number of units sold. Expenses can often be determined by summing fixed costs and variable cost. Variable cost is often expressed as variable cost per unit multiplied times the number of units. Thus, we can also express profit in the following mathematical model:

$$\begin{aligned}
\text{Profit} &= \text{Revenue} - (\text{Fixed cost} + \text{Variable cost}) \\
\text{Profit} &= (\text{Selling price per unit})(\text{Number of units sold}) \\
&\quad - [\text{Fixed cost} + (\text{Variable cost per unit})(\text{Number of units sold})] \\
\text{Profit} &= sX - [f + vX] \\
\text{Profit} &= sX - f - vX \qquad \textbf{(1-1)}
\end{aligned}$$

where

$$\begin{aligned}
s &= \text{selling price per unit} \\
f &= \text{fixed cost} \\
v &= \text{variable cost per unit} \\
X &= \text{number of units sold}
\end{aligned}$$

The parameters in this model are f, v, and s, as these are inputs that are inherent in the model. The number of units sold (X) is the decision variable of interest.

EXAMPLE: PRITCHETT'S PRECIOUS TIME PIECES We will use the Bill Pritchett clock repair shop example to demonstrate the use of mathematical models. Bill's company, Pritchett's Precious Time Pieces, buys, sells, and repairs old clocks and clock parts. Bill sells rebuilt springs for a price per unit of \$8. The fixed cost of the equipment to build the springs is \$1,000. The variable cost per unit is \$3 for spring material. In this example,

$$\begin{aligned}
s &= 8 \\
f &= 1{,}000 \\
v &= 3
\end{aligned}$$

The number of springs sold is X, and our profit model becomes

$$\text{Profit} = \$8X - \$1{,}000 - \$3X$$

If sales are 0, Bill will realize a \$1,000 loss. If sales are 1,000 units, he will realize a profit of \$4,000 (\$4,000 = (\$8)(1,000) − \$1,000 − (\$3)(1,000)). See if you can determine the profit for other values of units sold.

The BEP results in $0 profits.

In addition to the profit models shown here, decision makers are often interested in the **break-even point** (BEP). The BEP is the number of units sold that will result in $0 profits. We set profits equal to $0 and solve for X, the number of units at the BEP:

$$0 = sX - f - vX$$

This can be written as

$$0 = (s - v)X - f$$

Solving for X, we have

$$f = (s - v)X$$

$$X = \frac{f}{s - v}$$

This quantity (X) that results in a profit of zero is the BEP, and we now have this model for the BEP:

$$\text{BEP} = \frac{\textit{Fixed cost}}{(\textit{Selling price per unit}) - (\textit{Variable cost per unit})}$$

$$\text{BEP} = \frac{f}{s - v} \qquad \textbf{(1-2)}$$

For the Pritchett's Precious Time Pieces example, the BEP can be computed as follows:

$$\text{BEP} = \$1{,}000/(\$8 - \$3) = 200 \text{ units, or springs, at the BEP.}$$

The Advantages of Mathematical Modeling

There are a number of advantages of using mathematical models:

1. Models can accurately represent reality. If properly formulated, a model can be extremely accurate. A valid model is one that is accurate and correctly represents the problem or system under investigation. The profit model in the example is accurate and valid for many business problems.
2. Models can help a decision maker formulate problems. In the profit model, for example, a decision maker can determine the important factors or contributors to revenues and expenses, such as sales, returns, selling expenses, production costs, and transportation costs.
3. Models can give us insight and information. For example, using the profit model from the preceding section, we can see what impact changes in revenues and expenses will have on profits. As discussed in the previous section, studying the impact of changes in a model, such as a profit model, is called *sensitivity analysis*.
4. Models can save time and money in decision making and problem solving. It usually takes less time, effort, and expense to analyze a model. We can use a profit model to analyze the impact of a new marketing campaign on profits, revenues, and expenses. In most cases, using models is faster and less expensive than actually trying a new marketing campaign in a real business setting and observing the results.
5. A model may be the only way to solve some large or complex problems in a timely fashion. A large company, for example, may produce literally thousands of sizes of nuts, bolts, and fasteners. The company may want to make the highest profits possible given its manufacturing constraints. A mathematical model may be the only way to determine the highest profits the company can achieve under these circumstances.
6. A model can be used to communicate problems and solutions to others. A decision analyst can share his or her work with other decision analysts. Solutions to a mathematical model can be given to managers and executives to help them make final decisions.

Mathematical Models Categorized by Risk

Deterministic means with complete certainty.

Some mathematical models, like the profit and break-even models previously discussed, do not involve risk or chance. We assume that we know all values used in the model with complete certainty. These are called **deterministic models**. A company, for example, might want to minimize